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Usability of explicit filtering in large eddy simulation with a low-order numerical scheme and different subgrid-scale models

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SUMMARY

Based on *a priori* tests, in large eddy simulation (LES) of turbulent fluid flow, the numerical error related to low-order finite-difference-type methods can be large in comparison with the effect of subgrid-scale (SGS) model. Explicit filtering has been suggested to reduce the error, and it has shown promising results in *a priori* studies and in some simulations with fourth-order method. In this paper, the effect of explicit filtering on the total simulation error is studied together with a second-order scheme, where the numerical error should be even larger. The fully developed turbulent channel flow between two parallel walls is used as a test case. Rather simple SGS models are applied, because these models are most likely used in practical applications of LES. Explicit filtering is here applied to the non-linear convection term of the Navier–Stokes equations, four three-dimensional filter functions are applied, and the effect of filtering is separated from the effect of SGS modelling. It is shown that the effect of filtering is rather large and smooth filters introduce an additional error component that increases the total simulation error. Finally, filtering *via* subfilter-scale modelling is applied, and it is shown that this approach performs better. However, the large-frequency components of the resolved flow field are not as effectively damped as when the non-linear convection term is filtered. Copyright © 2007 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In large eddy simulation (LES), the large scales of turbulent fluid motion are solved from the filtered Navier–Stokes equations, and a subgrid-scale (SGS) or subfilter-scale (SFS) model is applied to describe the effect of the small scales on the large ones. When finite-difference-type methods,

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where discretization is not performed in the spectral space, are applied and the computational grid is let to define the separation between resolved and SGS scales, the smallest resolved scales are of the same size as the grid resolution. Low-order (i.e. second or fourth order) schemes are often used in LES applications, and with these schemes the numerical error becomes a problem. The numerical error has been evaluated in *a priori* studies, and it has been shown that the error can be larger than the effect of the SGS model [1, 2]. However, according to *a priori* tests, explicit filtering of the resolved flow field seems to be one promising approach to improving the situation [1, 2]. The aim of this paper is to further evaluate this approach.

Filtering of the whole velocity field in the end of each time step has been the traditional approach to explicit filtering [3], but if a non-sharp filter is applied, it leads to multiple filtering of the velocity field from the previous time levels [4], and in *a priori* tests, it has led to unphysical behaviour of the SGS term [5]. Explicit filtering of the non-linear convection term of the Navier–Stokes equations has been suggested as an alternative approach [4]. The approach has been successfully applied in actual simulations using a fourth-order discretization method [6–8], and it gave promising results also for a second-order scheme in *a priori* tests [5]. A totally different approach to filtering was proposed in Reference [9] where filtering is performed *via* SGS and SFS modelling. When explicit filtering is applied, there are scales that are smaller than the filter width but larger than the grid spacing, i.e. SFSs, and they also need modelling. In the approach of Reference [9], the SGS and SFS stresses are modelled separately and the actual filtering operation is present only in the SFS model. This approach has been applied with several SFS models in References [10, 11].

In this paper, the effect of the above-mentioned explicit filtering approaches on the total simulation error, i.e. the difference in DNS (direct numerical simulation) data, is studied. A fully developed turbulent channel flow between two parallel walls is used as a test case. These studies are performed mainly with the standard Smagorinsky model, because it is a rather simple SGS model, which is still widely applied in practical applications. However, it is also verified that the behaviour of explicit filtering is similar also with some more advanced models. The second-order discretization scheme is chosen here because, according to *a priori* tests, the related numerical error is quite large [1, 2]; thus, the presumable effect of explicit filtering on this error should be clear.

This paper is organized as follows. The numerical methods, filter functions and SGS models are discussed in Section 2. Explicit filtering of the convection term is studied in Sections 3–5. In Section 3, different explicit filter functions are applied in simulations with the standard Smagorinsky model. Some observations of the results are made and the effect of the shape of the filter function on these results is studied. To further evaluate the results of Section 3, the effect of SGS model and explicit filtering are separated in Section 4. This is done by repeating the simulations with the Smagorinsky model with different model length scales and also without explicit filtering. First, the effect of filtering is obtained by studying cases with and without filtering but with no SGS model. Then, the effect of filtering on the SGS model is obtained by comparing results with SGS model between cases with and without filtering. In Section 5, the simulations are repeated using also the dynamic Smagorinsky, scale similarity and mixed models to verify that the results of the previous sections are not caused by poor SGS modelling. To demonstrate the effect of SGS model and the effect of filtering on the different models, the simulations are repeated both with and without explicit filtering. In Section 6, explicit filtering of the convection term is compared with filtering *via* SFS modelling. The need for SFS modelling and reconstruction of the SFS stresses is demonstrated by using first only Smagorinsky models, which provide a model for the SGS stresses, and then the scale-similarity model to include also the SFS stresses. These results

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are compared with the results with filtering of the convection term. Finally, conclusions on these filtering approaches are drawn in Section 7.

2. APPLIED NUMERICAL METHODS AND MODELS

2.1. Test case and governing equations

In the present study, a fully developed turbulent channel flow between two infinite parallel walls is taken as the test case. The Reynolds number is $Re_\tau = 395$ based on the channel half-height and the friction velocity $u_\tau = \sqrt{\tau_{\text{wall}}/\rho}$, where τ_{wall} is the wall shear stress and ρ the density.

The filtered Navier–Stokes equations that are being solved in LES are expressed here in the non-dimensional form as

$$\frac{\partial \tilde{u}_i}{\partial t} = -\frac{\partial P}{\partial x_i} - \frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(-\tilde{u}_i\tilde{u}_j - \tau_{ij} + \frac{1}{Re_\tau}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)\right) \tag{1}$$

where $(x_1, x_2, x_3) = (x, y, z)$ refer to non-dimensional streamwise, wall-normal and spanwise spatial coordinates, respectively, t to time, $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3) = (u, v, w)$ to resolved velocity vector, P to mean pressure, $\tilde{p}$ to fluctuating resolved pressure and τ_{ij} is a model for the SGS stress tensor $\widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$. Here, the equations are scaled by the channel half-height, $0.5h$, and friction velocity, u_τ.

Traditionally, when Equations (1) are solved numerically, no explicit filtering is present, and the reduction to a discrete grid and the applied numerical scheme and SGS model are interpreted as implicit filtering. The approaches applied to explicit filtering in this paper are discussed in the next section.

In the present simulations, the second-order central-difference scheme is applied on a staggered grid system [12] for spatial discretization, and for time integration, a third-order, three-stage Runge–Kutta method [13] is applied. The non-dimensional mean-pressure gradient is fixed, and the fluctuating pressure is solved from a Poisson equation. The applied grid resolution and the dimensions of the computational domain are given in Table I.

2.2. Approaches to filtering

In traditional LES, the reduction to a discrete grid and the numerical differentiation scheme are interpreted as implicit filtering, and no filtering operation is explicitly present in the simulation. However, in the equations being solved in LES (1), the non-linear convection term, $\tilde{u}_i\tilde{u}_j$, introduces high-frequency components to the resolved flow field $\tilde{u}_i$. The idea in explicit filtering is to insure

Table I. Applied grid resolution and dimensions of the computational domain.

	x	z	y
Extent of the domain (scaled by h)	6.0	3.2	1.0
Number of grid points	108	108	90
Size of the grid cell in wall units (Δ^+)	44	23	min 1.0, max 20

Note: Wall units: $x^+ = Re_\tau x$, where x is scaled by the channel half-height.

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that the frequency content of all the terms in the equations is the same and to explicitly remove the high frequencies that are badly described by the discrete grid.

The most straightforward approach to implement explicit filtering would be to filter the resolved velocity field $\tilde{u}_i$ at the end of each time step as done in [3], where the spectral cutoff filter was applied in the homogeneous directions of the turbulent channel flow. However, when finite-difference-type methods are applied, filtering is usually performed in the physical space and the filter function is not a projection like the spectral cutoff filter, i.e. filtering a quantity twice further damps down the high frequencies. This is a problem especially when an explicit time-integration method is applied [4].

Since the non-linear convection term of the Navier–Stokes equations is the only term that introduces high-frequency components into the resolved flow field, it is actually sufficient to filter only this term [4]. In these approaches, the momentum equations being solved are expressed as

$$\frac{\partial \tilde{u}_i}{\partial t} = -\frac{\partial}{\partial x_j}(\overline{\tilde{u}_i \tilde{u}_j} + \bar{\tau}_{ij}) - \frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\frac{1}{Re_\tau}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)\right) \tag{2}$$

where the overbar again means explicit filtering. In this approach, the definition of the SGS term is expressed as $\widetilde{u_i u_j} - \overline{\tilde{u}_i \tilde{u}_j}$, which differs from the definition of Equation (1). If the SGS model is not purely dissipative, it should also be filtered explicitly as expressed in Equation (2). However in the present study, this did not have a significant effect on the obtained flow statistics.

If only the non-linear convection term is filtered, the Leonard stress is included implicitly in the convective term [6]. This approach to explicit filtering has some similarities to the Leray modelling of Reference [14] where the convection velocity is filtered. In this paper, we mainly focus on the filtering of the non-linear convection term.

In Equation (2), notation $\tilde{u}_i$ is used for the resolved velocity field. However, the resolved velocity field is affected by the filtering and thus it is not the same as the resolved velocity of Equation (1) where no explicit filtering is applied.

There are also alternative approaches to explicit filtering which stress the role of modelling. When explicit filtering is applied, in addition to SGS stresses, there are SFS stresses that also require modelling. An alternative formulation for explicit filtering where filtering and discretization processes are distinguished, and the SGS and SFS stresses are modelled separately, was proposed in [9]. In this formulation, the actual filtering operation is performed only in the SFS model. The filtered Navier–Stokes equations are expressed as

$$\frac{\partial \bar{\tilde{u}}_i}{\partial t} = -\frac{\partial}{\partial x_j}(\widetilde{\bar{\tilde{u}}_i \bar{\tilde{u}}_j} + \widetilde{T}_{ij}) - \frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\frac{1}{Re_\tau}\left(\frac{\partial \bar{\tilde{u}}_i}{\partial x_j} + \frac{\partial \bar{\tilde{u}}_j}{\partial x_i}\right)\right) \tag{3}$$

where 'tilde' refers to reduction to the discrete grid and the notation $\bar{\tilde{u}}_i$ stresses the indirect effect of explicit filtering on the resolved velocity field. T_{ij} represents both SGS and SFS stresses and it is defined as

$$\begin{aligned} T_{ij} &= \overline{u_i u_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = (\overline{u_i u_j} - \overline{\tilde{u}_i \tilde{u}_j}) + (\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j) \\ &= \bar{\tau}_{ij} + (\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j) \end{aligned} \tag{4}$$

where the first term is the SGS stress and the second one the SFS stress.

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The clear difference of the two discussed approaches is that in the formulation of Equations (3) and (4), the actual explicit filtering appears only in the SFS model. The simplest possible SFS model for the last term of Equation (4) would be to approximate $\tilde{u}_i$ by $\overline{\tilde{u}}_i$, which means using the scale-similarity model (SSM). The approach of [9] was applied in [11] with a tensor-diffusivity model and in [10] with several SFS models. In [11], it was noticed that in the turbulent channel flow, the results were similar to the ones obtained with no filtering and in [10] the simulation results were improved when compared with a case with no filtering when a high-order reconstruction was applied to the SFS stresses. In both studies, the results with explicit filtering were compared with a simulation with the dynamic Smagorinsky model (DSM) with the spectral cutoff as the test filter.

2.3. *Explicit filters*

In this paper, explicit filtering operation is applied in the physical space, and it is expressed as

$$\overline{\tilde{u}_i(x_j)\tilde{u}_i(x_j)} = \sum_{l=-K}^{K} a_l \tilde{u}_i(x_{j+l})\tilde{u}_i(x_{j+l}) \tag{5}$$

where $\bar{\cdot}$ is the filtered quantity, x_j is the position in space, a_l are the coefficients of the discrete filter and K is the number of grid points applied by the filter. In this paper, four discrete filters with the widths of two grid spacings ($\Delta_\mathrm{f} = 2\Delta$) are applied: the Simpson, trapezoidal and two commutative filters. Here, the filter width is the same in all test cases, and filtering is applied in all the three spatial directions.

The filter transfer functions of the applied filters are depicted in Figure 1 in the spectral space. The so-called spectral cutoff filter is included for comparison. The Simpson and trapezoidal filters are discrete approximations to the top-hat filter, which is defined in the physical space. The commutative filters are discrete approximations to the spectral cutoff filter. Their advantage is that the error due to changing the order of filtering and differentiation, which appears on stretched grids, is reduced [15]. While the Simpson and trapezoidal filters have second-order commutation error, the commutative filters applied here have fourth- and sixth-order errors. In the present test case, this error is present only in the wall-normal direction where the grid is stretched. The coefficients of one-dimensional filters are given in Table II, and the three-dimensional filter is obtained by applying filtering sequentially to each coordinate direction.

Besides the commutation error, there are two main differences between these filters: the shapes of the filter functions and the effective filter widths. The shape of the filter transfer function affects the error in the conservation of kinetic energy and in the Galilean invariance, which are broken in this formulation of explicit filtering if a filter that differs from the spectral cutoff filter is applied [4]. Of the applied filters, the sixth-order commutative filter is closest to the spectral cutoff filter; thus, these errors should be the smallest with this filter. In the high-frequency part, the Simpson filter and in the low-frequency area, the trapezoidal filter are furthest away from the spectral cutoff. Due to its shape, the trapezoidal filter damps down the low frequencies more than the other filters, while the Simpson filter does not damp down the high frequencies as much as the other filters. The effective filter widths of the commutative and Simpson filters are two grid spacings, but the width of the trapezoidal filter is 2.45 [4].

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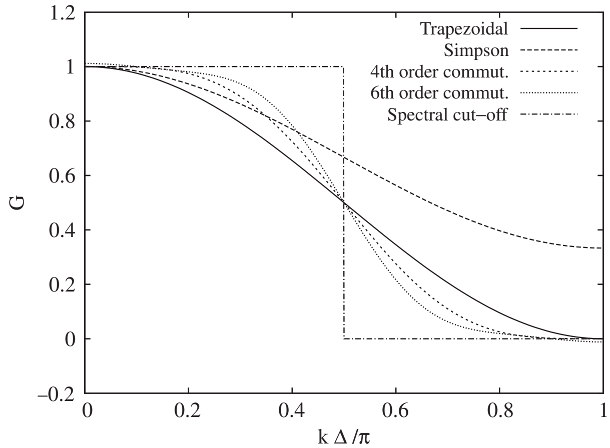

Figure 1. Transfer function $\widehat{G}$ of Simpson, trapezoidal and two commutative filters.

Table II. Coefficients of discrete filter with $\Delta_\mathrm{f}=2\Delta$.

	a_0	$a_{\pm 1}$	$a_{\pm 2}$	$a_{\pm 3}$	$a_{\pm 4}$	$a_{\pm 5}$
S	$\frac{2}{3}$	$\frac{1}{6}$				
T	$\frac{1}{2}$	$\frac{1}{4}$				
C1	$\frac{1}{2}$	$\frac{9}{32}$	0	$\frac{-1}{32}$		
C2	$\frac{1}{2}$	$\frac{75}{256}$	0	$\frac{-25}{512}$	0	$\frac{3}{256}$

S, Simpson; T, Trapezoidal; C1, fourth-order commutative filter; C2, sixth-order commutative filter.

2.4. SGS modelling

In this paper, the standard [16] and dynamic versions [17] of the Smagorinsky model and a SSM [18] are applied to model the SGS shear stress. In Smagorinsky models, the eddy-viscosity concept is applied and τ_{ij} is expressed as

$$-\tau_{ij}=\mu_T 2S_{ij}=\underbrace{(C_\mathrm{S}\Delta_\mathrm{S})^2\sqrt{2S_{ij}S_{ij}}}_{=\mu_T}\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right) \tag{6}$$

where C_S is the model parameter and Δ_S the model length scale. In this study, Δ_S is set equal to grid spacing as

$$\Delta_\mathrm{S}=(\Delta x\Delta y\Delta z)^{1/3} \tag{7}$$

or, in cases where explicit filtering is applied, to the explicit filter width, Δ_f. Explicit filtering reduces the effective resolution of the grid, and it is necessary to model the effect of the length scales that are smaller than the filter width instead of scales smaller than the grid spacing. In the Smagorinsky models, the size of the largest modelled scales is controlled *via* the model length scale [19], and this is why the model length scale is here varied with the filter width.

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When the standard version of the model is applied, the value 0.085 is used for C_S, and the value is reduced near the solid walls using the van Driest damping. In the dynamic version of the model, C_S is evaluated using the procedure proposed by Lilly [20]. In the dynamic model, a test filter is required in the evaluation of C_S. When no explicit filtering is applied, a three-dimensional fourth-order commutative [15] test filter with the width of two grid spacings is used, and when explicit filtering is applied, a test filter with the width of four grid spacings, i.e. twice the explicit filter width, is applied. The test filter depends on explicit filtering in the same way as the model coefficient of the standard version of the model.

In the SSM, the SGS shear stress is modelled as

$$\tau_{ij} = \overline{\tilde{u}_i \tilde{u}_j} - \overline{\tilde{u}}_i \overline{\tilde{u}}_j \tag{8}$$

where the overbar refers to a second filter that is applied explicitly. The same three-dimensional filter as applied in the DSM is applied here. When explicit filtering is applied, the SSM provides a first-order reconstruction for the SFS that are scales smaller than the explicit filter width but larger than the gird spacing. Also, a mixed model (MM) [21], where SSM models the SFS stress and a Smagorinsky model models the SGS stress, is applied.

3. EFFECT OF FILTER SHAPE

In this section, the non-linear convection term of the Navier–Stokes equations is filtered explicitly (Equation (2)) using different filter functions, Simpson, trapezoidal and fourth- and sixth-order commutative filters, which were discussed in Section 2.3. The aim of the section is to describe the effect of the differences between the filters on the simulation results.

The standard Smagorinsky model is applied to SGS modelling. In cases with explicit filtering, the model length scale, Δ_S, is set equal to the filter width (here two grid spacings), and in the case with no filtering, the model length scale is equal to grid spacing. This choice was made because the SGS model should model the effect of scales smaller than the filter width; thus, in cases with explicit filtering, the effect of the SGS model must be increased. The larger effective filter width of the trapezoidal filter was not taken into account, and the SGS model length scale was the same with all the filters.

The mean-velocity profiles obtained using different discrete filters are plotted in Figure 2 and the streamwise deviatoric Reynolds stress component,

$$\langle u'u' \rangle^d = \langle u'u' \rangle - \tfrac{1}{3}(\langle u'u' \rangle + \langle v'v' \rangle + \langle w'w' \rangle) \tag{9}$$

where the brackets refer to the average over homogeneous directions and time, is given in Figure 3. Only the deviatoric part is studied as suggested in [22] since the Smagorinsky model is traceless. All the filters increase the total simulation error when compared with the case with no explicit filtering. The total error is smaller with the Simpson filter than with the trapezoidal filter, whereas there are no essential differences between the two commutative filters.

The one-dimensional energy spectra E_{uu} for the streamwise velocity component from the near-wall region and from the middle of the channel are evaluated in the streamwise direction in Figures 4 and 5, respectively. The differences between the filters are quite small especially in the middle of the channel. The Simpson filter transfer function obtains quite large values near the grid cutoff (see Figure 1), but still it damps the high frequencies as efficiently as the other filters.

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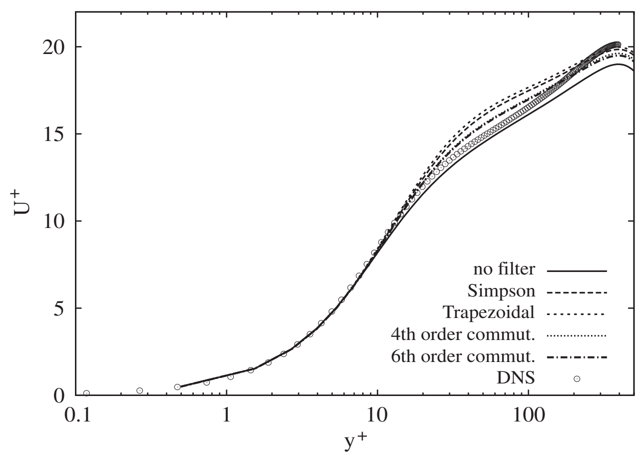

Figure 2. Comparison between filter functions. Mean-velocity profile.

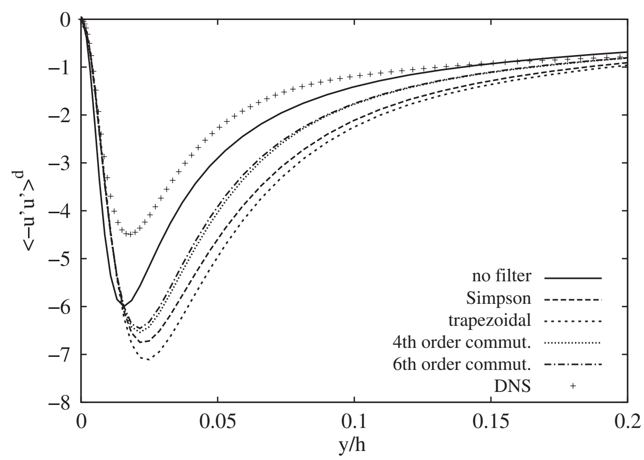

Figure 3. Comparison between filter functions. Deviatoric streamwise Reynolds stress.

The τ_{12} component of the SGS shear stress (see Equation (6)) is depicted in Figure 6. It is mainly controlled *via* the model length scale; thus, the increment when compared with the case with no filtering is understandable. The choice of the filter function has only a small effect on this quantity.

On the basis of the results in this section, one can say that although explicit filtering should decrease the numerical error, it increased the total simulation error. Despite the differences between the applied filter functions, the choice of the filter function has quite a small effect on the flow statistics. The commutative filters produce somewhat better results than the Simpson or trapezoidal filters. The sixth-order filter requires slightly more computational effort than the fourth-order filter, but there are no essential differences between the two filters and thus, for the rest of paper, the fourth-order filter is applied as the explicit filter.

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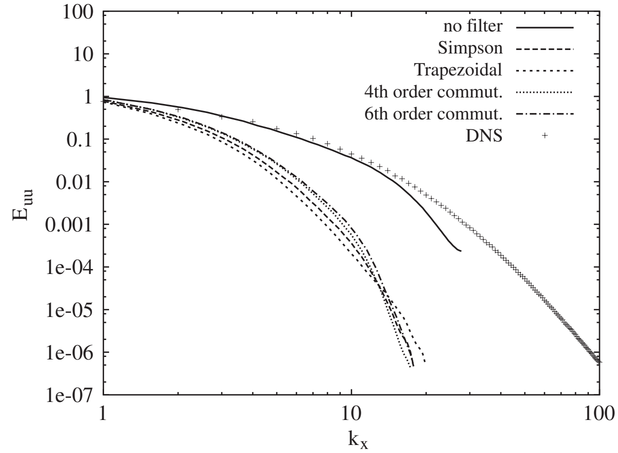

Figure 4. Comparison between filter functions. One-dimensional energy spectra E_{uu} in the near-wall region. Streamwise direction.

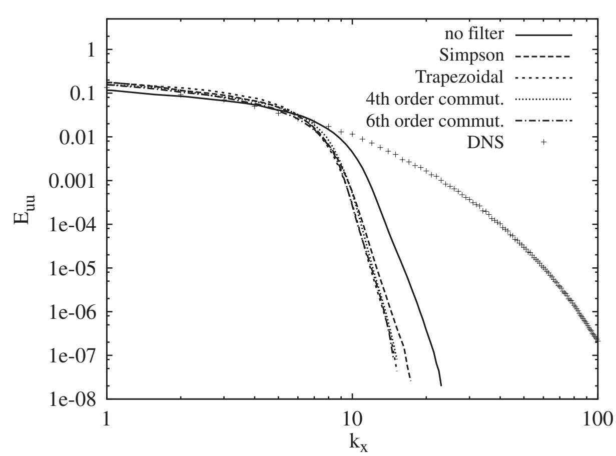

Figure 5. Comparison between filter functions. One-dimensional energy spectra E_{uu} in the middle of the channel. Streamwise direction.

4. EFFECT OF FILTERING *VERSUS* EFFECT OF SGS MODELLING

In the previous section, the Smagorinsky length scale, Δ_S, was kept equal to the filter width, Δ_f, i.e. two grid spacings, in all cases with explicit filtering, and it was equal to the grid spacing in the cases with no filtering. Thus, in cases with explicit filtering, the quality of the results was a combination of filtering and modelling. In this section, the aim is firstly to describe the effect of mere filtering on the simulation results. Secondly, we wish to compare the effect of modelling in cases with and without explicit filtering. Thirdly, the interaction between modelling and filtering is demonstrated.

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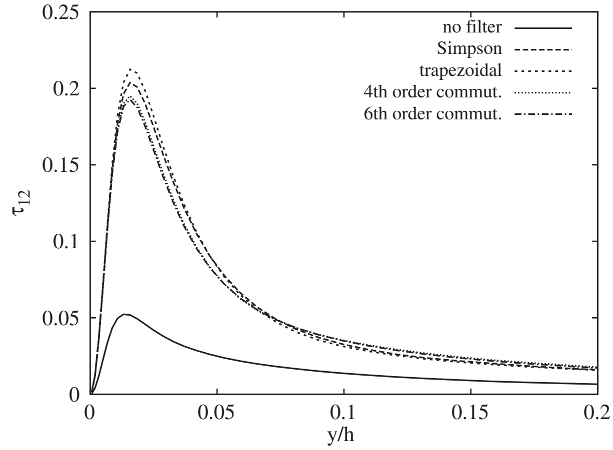

Figure 6. Comparison between filter functions. SGS shear stress τ_{12}.

To separate the effect of filtering, we compare the results from simulations with and without filtering using no SGS model. The effect of model on filtered results is seen by studying results with filtering and different model length scales. When the model length scale is increased and explicit filtering is applied, the behaviour of the model is not the same as in the case with no filter. This effect of filter on the model is studied by repeating the simulations with different length scales using no filtering.

In this section, in all cases with explicit filtering, the filter is the fourth-order commutative filter with the width of two grid spacings and the non-linear convection term is filtered. The only parameter being varied is the model length scale.

In the upper part of Figure 7, we have the mean-velocity profiles from simulations with explicit filtering, and in the lower part, from simulations without explicit filtering. In both cases, the simulations were repeated with different values of the model length scale Δ_S. If we compare the two figures, we notice that, when explicit filtering is applied, the effect of modelling is reduced, i.e. the difference to the case with no model ($\Delta_S = 0$) reduces. We notice that most of the changes obtained in the previous section when explicit filtering was applied are already present in the case with no model in the upper part of the figure. Thus, the changes were mainly the result of filtering itself. However, we further see in the same figure that, when the Smagorinsky length scale is increased, the prediction of the mean-velocity profile slightly improves. The viscous sublayer becomes better captured and the slope of the profile improves slightly. Thus, the model is able to compensate somewhat for the effect of filtering.

In Figure 8, corresponding plots are given for the streamwise deviatoric Reynolds stress. Most of the overprediction of the Reynolds stress that was noticed in the previous section is already present in the case with no model and results thus from mere filtering. When the model length scale is increased, the peak value reduces and moves towards the middle of the channel. As seen in the lower part of the figure, this is owing to modelling, and in the upper part, the use of explicit filtering prevents the profile from widening too much. The results are similar for the other Reynolds stress components.

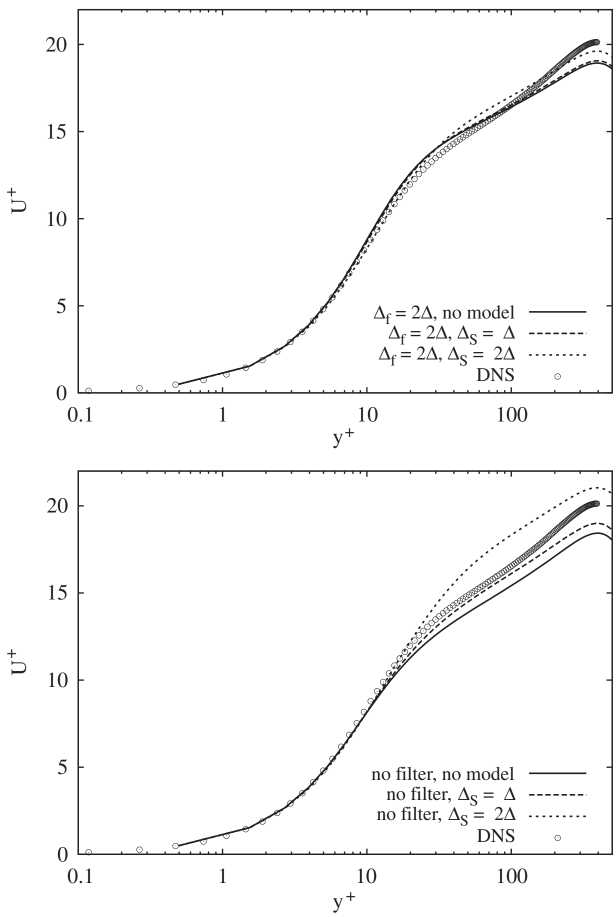

Figure 7. Effect of model and filtering. Mean-velocity profile. Upper: Explicit filtering is applied with the filter width of two grid spacings. Lower: No explicit filtering is applied.

The one-dimensional energy spectra in the streamwise direction are depicted in the near-wall region in Figure 9. This location was chosen because it demonstrates quite clearly the effect of both modelling and filtering. We notice that the main difference between explicit filtering and the damping provided by the model is their effect on the high frequencies. The model affects the low and high frequencies in a similar manner, whereas explicit filtering damps the high frequencies much stronger than the low frequencies. Also other authors have noticed that eddy-viscosity-type models can affect a wider range of scales than explicit filtering [23, 24].

In the previous sections, we noticed that the total error increased when explicit filtering was applied, and the choice of the filter function did not have a large effect on this. Based on the results of this section, there seems to be two factors behind this behaviour: the effect of filtering itself and the behaviour of the applied SGS model with filtering. With the current numerical methods, explicit filtering alone had a large affect on the flow statistics and it was the major factor behind the large total error. This new error introduced by explicit filtering overshadows the possible positive

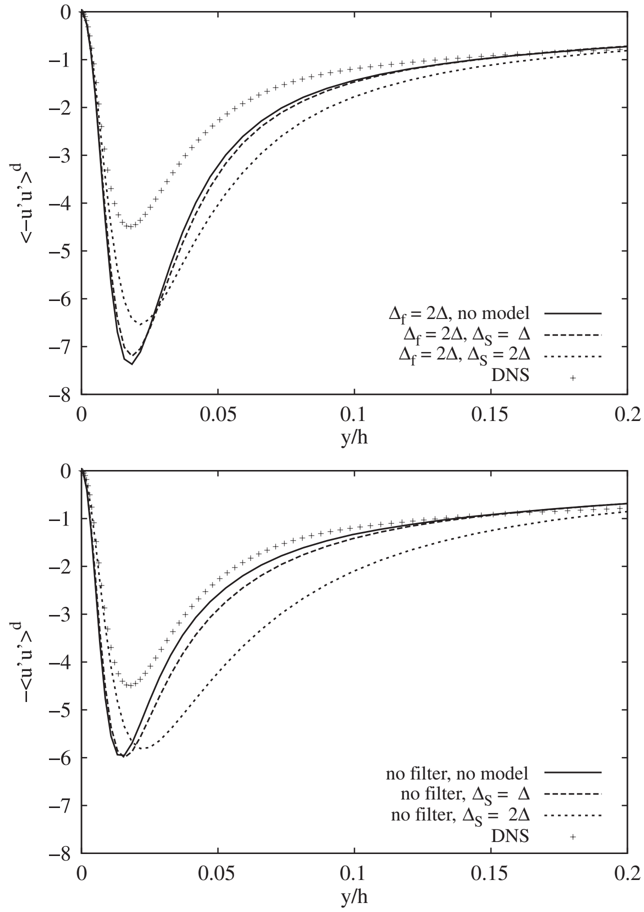

Figure 8. Effect of model and filtering. Deviatoric streamwise Reynolds stress. Upper: Explicit filtering is applied with the filter width of two grid spacings. Lower: No explicit filtering is applied.

effect filtering might have on the numerical error. When SGS modelling was applied, filtering decreased the effect of modelling, but as the model length scale was increased, the model was able to improve the prediction of the flow statistics compared with the case with no model. In addition, in cases with the large model length scale, the results with explicit filtering were better than the results without explicit filtering. Thus, despite the large effect of filtering itself on the total simulation error, there was some desirable interaction between filtering and SGS modelling.

5. EXPLICIT FILTERING AND SOME SGS MODELS

The results of the previous subsections were obtained using the standard Smagorinsky model, which is one of the simplest SGS models. It is a dissipative model that is usually used to model SGS stresses. In the previous sections, no separate modelling or reconstruction was applied to the

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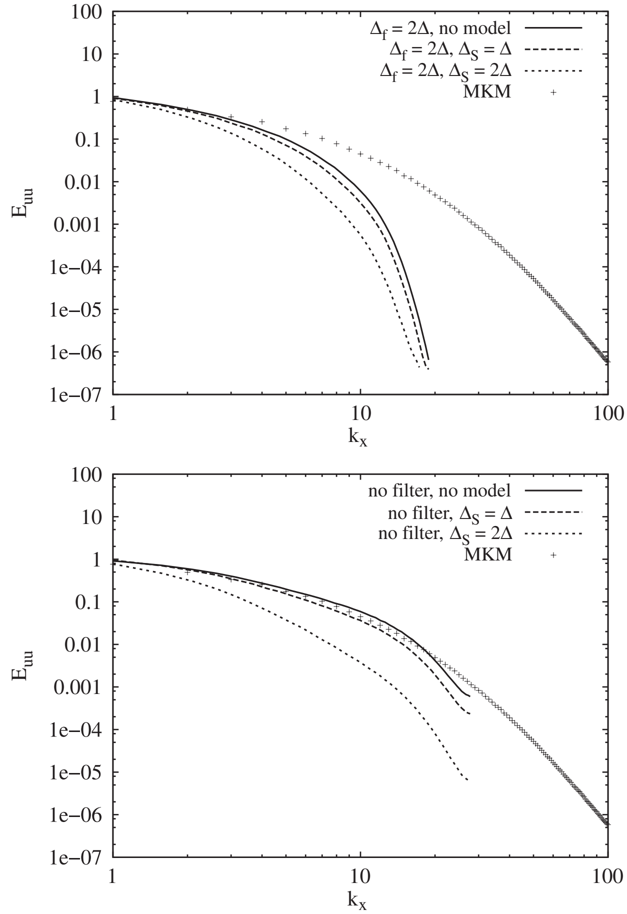

Figure 9. Effect of model and filtering. One-dimensional energy spectra. Streamwise direction. Upper: Explicit filtering is applied with the filter width of two grid spacings. Lower: No explicit filtering.

SFS stresses, which are related to scales smaller than filter width but larger than grid spacing. To verify that the results were not caused by poor SGS or SFS modelling, these simulations are repeated here using the dynamic version of the Smagorinsky model (DSM) for the SGS stresses, the SSM for the SFS stresses and a MM, where the Smagorinsky model is used to model SGS stresses and SSM to model SFS stresses. To further demonstrate the effect of explicit filtering on model, the simulations with these models are presented both with and without explicit filtering.

In the simulations of this section, the fourth-order commutative filter with the width of two grid spacings is applied as the explicit filter as well as the filter of the SSM. In simulations with explicit filtering, the test filter of the dynamic model is the commutative filter with the width of four grid spacings, and in simulations with no explicit filtering the test filter is the commutative filter with the width of two grid spacings. In simulations with explicit filtering and the standard Smagorinsky model, the length scale of the Smagorinsky models is set equal to the filter width.

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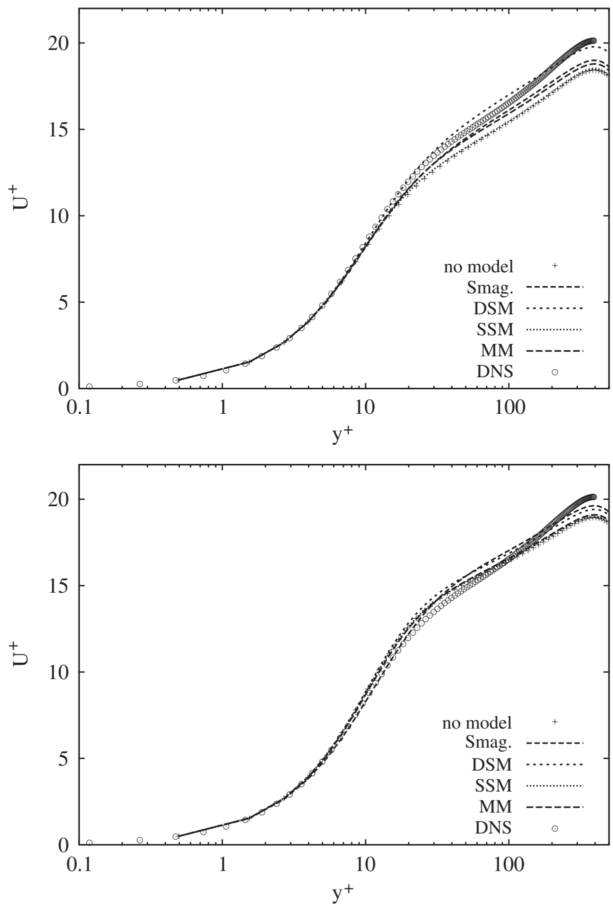

Figure 10. Mean-velocity profile. Explicit filtering of the convection term with different SGS models. Upper: No explicit filtering. Lower: Explicit filtering with $\Delta_f = 2\Delta$.

In simulations with the MM, the length scale of the Smagorinsky model was equal to grid spacing, because the scale-similarity part models the SFSs.

The mean-velocity profiles from simulations with different models are given in Figure 10. In the upper part of the figure, no explicit filtering is applied, and in the lower part, the non-linear convection term and the SGS and SFS models are filtered explicitly using the filter width of the two grid spacings. When no explicit filtering is applied, there are clear differences between the profiles obtained using different models, and the DSM produces the value closest to the DNS data.

In the lower part of Figure 10, we notice that explicit filtering reduces the differences between the models and the curves are closer to the case with no modelling, meaning that the effect of modelling is decreased. As noticed in the previous section with the standard Smagorinsky model, filtering increases the total error. Here, we notice that despite the use of somewhat better models and inclusion of the first-order reconstruction for the SFS stresses, the slope still changes by the same amount.

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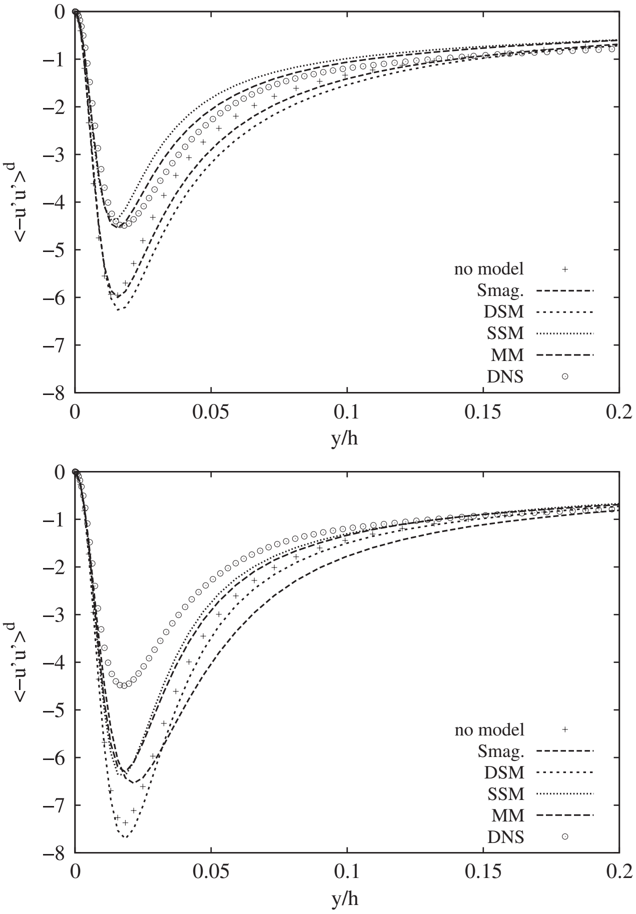

Figure 11. Resolved deviatoric streamwise Reynolds stress. Explicit filtering of the convection term with different SGS models. Upper: No explicit filtering. Lower: Explicit filtering with $\Delta_f = 2\Delta$.

The resolved deviatoric streamwise Reynolds stress obtained with different models is plotted in Figure 11. Again, we notice that despite the improved modelling, the total error increases and the effect of modelling decreases when explicit filtering is applied.

The SGS or SFS shear stresses, τ_{12}, obtained with and without explicit filtering are plotted in the upper and lower parts of Figure 12, respectively. We notice that with DSM, SSM and MM, τ_{12} clearly decreases when filtering is applied, which is in agreement with the results obtained for the mean-velocity profile and for the Reynolds stress. With the standard Smagorinsky model, τ_{12} increases because of the increased model length scale.

The one-dimensional energy spectra from the near-wall region ($y^+ \approx 5$) are evaluated in the streamwise direction in Figure 13. In the upper part of the figure where no explicit filtering is applied, the differences between the models are rather small, whereas in the lower part, the standard Smagorinsky model damps down the spectrum most. It was noticed in the previous section that damping of the low frequencies mainly results from the implicit filtering provided by the model.

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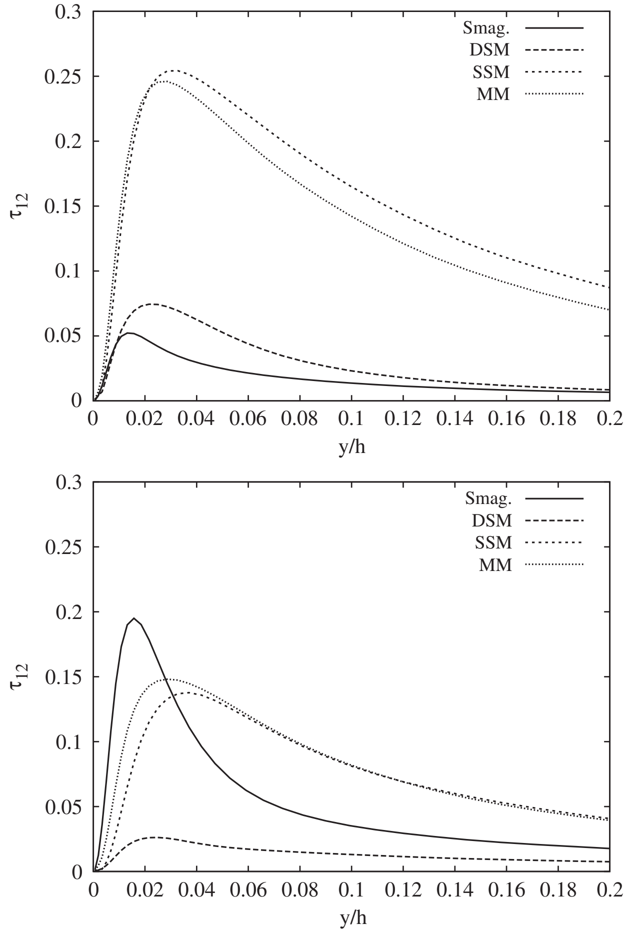

Figure 12. SGS shear stress. Explicit filtering of the convection term with different SGS models. Upper: No explicit filtering. Lower: Explicit filtering with $\Delta_{\mathrm{f}} = 2\Delta$.

Here, we see that with SSM, which introduces no extra dissipation to the simulation, the low frequencies are least affected. The benefit of using a model or reconstruction for SFSs is also demonstrated here: when an SFS model is used as in SSM or MM, the damping of the low frequencies is diminished. When explicit filtering is applied, the high frequencies are efficiently damped with all the models, which suggests that although the total error was not decreased, the numerical error is probably smaller.

Based on the results of this section, it seems that although in SSM and MM a simple reconstruction is used to model the SFSs, the same deficiency as noticed with the Smagorinsky models is seen with these models when the non-linear convection term is filtered. In SSM, there is no direct interaction between the model and explicit filtering. In DSM, modelling and explicit filtering are coupled *via* the increased test-filter width. It has been noticed that as long as the width of the explicit filter is correctly treated, the results are not sensitive to the choice of the test filter [4].

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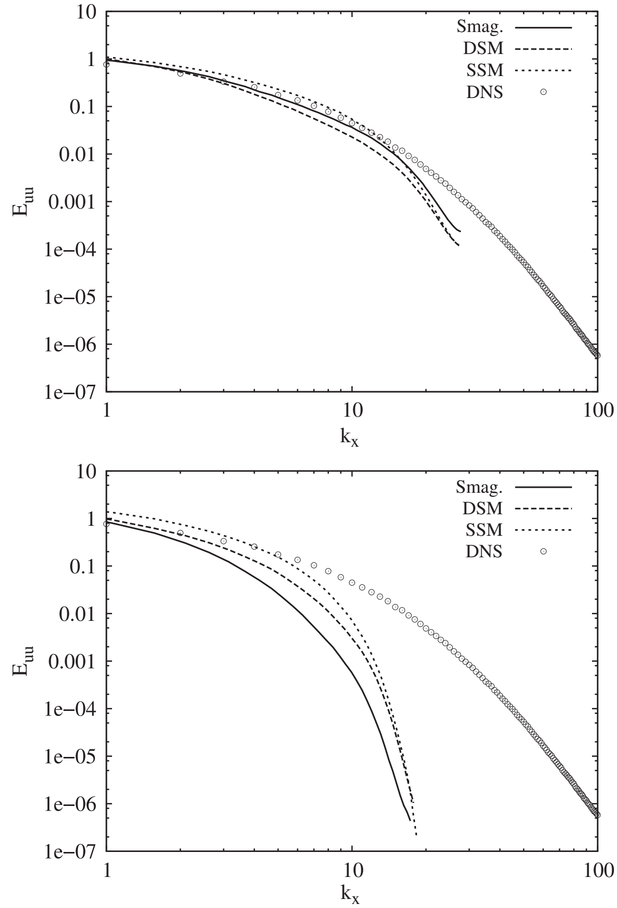

Figure 13. One-dimensional energy spectra at $y^+ \approx 5$. Streamwise direction. Explicit filtering of the convection term with different SGS models. Upper: No explicit filtering. Lower: Explicit filtering with $\Delta_f = 2\Delta$.

Since, in addition, explicit filtering reduces the effect of the model, it is understandable that the effect of the model did not increase. With the standard Smagorinsky model, the coupling between the model and filtering is easily set *via* the model length scale, but the modelling error limits the accuracy of the results.

6. SFS FILTERING *VERSUS* FILTERING OF THE CONVECTION TERM

In the previous sections, we saw that explicit filtering can have a large negative effect on the simulation results. In this section, we apply the approach presented in [9] where filtering is performed *via* SFS modelling (see Section 2.2). The same models as applied in the previous section are applied here, and the aim is to compare filtering of the non-linear convection term to filtering

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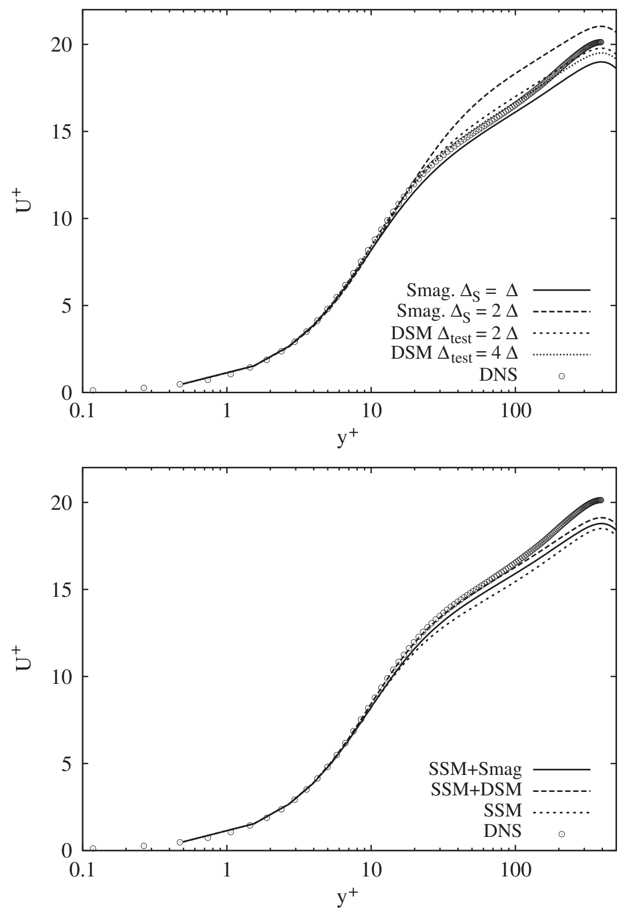

Figure 14. Mean-velocity profile. Filtering provided *via* the model. Upper: Only SGS stress is modelled. Lower: SFS or SFS and SGS stresses are modelled.

via SFS modelling. Here, modelling is first applied only to the SGS component of the shear stress using the Smagorinsky models, then only the SFS component is modelled using the SSM, and finally, both SFS and SGS stresses are modelled using SSM and a Smagorinsky model as a MM. It has been shown that the modelling of the SFS stress is necessary [10], and to obtain improved results compared with the traditional approach, the dynamic reconstruction model [25] had to be applied. Here, much simpler models are applied because the main goal is to compare the two approaches.

In the upper part of Figure 14, the mean-velocity profiles from the cases using a Smagorinsky model for the SGS stress are depicted. First, no extra filtering is provided *via* the model. The model length scale in the standard Smagorinsky model, Δ_S, is equal to the grid spacing, and the test filter in DSM, Δ_{test}, has the width of two grid spacings. Then, the model length scale in the standard Smagorinsky model is increased to two grid spacings and in DSM, the width of the test filter is increased to four grid spacings. This corresponds to a situation where the explicit filter has

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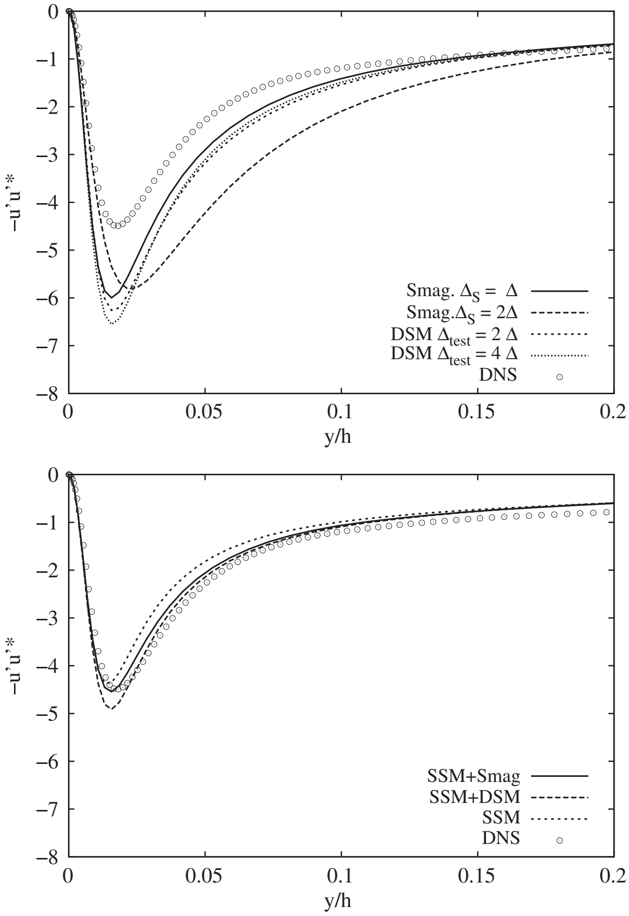

Figure 15. Resolved deviatoric streamwise Reynolds stress. Filtering provided *via* the models. Upper: Only SGS stress is modelled. Lower: SFS or SFS and SGS stresses are modelled.

the width of two grid spacings. In the standard Smagorinsky model, the filtering provided by the model is actually implicit filtering since no filtering operation is made explicitly. In Figure 14, the larger model length scale in the standard Smagorinsky model increases the thickness of the viscous sublayer and makes the mean bulk velocity overpredicted. In the DSM, increasing the test filter width has only a small effect on the velocity profile.

In the lower part of Figure 14, we have the velocity profiles from cases where SSM is used alone as an SFS model and as a MM together with Smagorinsky models to model both SGS and SFS stresses. The filter in SSM has the width of two grid spacings, and when the MM is applied, the model parameter in the standard Smagorinsky model is proportional to the grid spacing, and the test-filter width in DSM is two grid spacings. As seen in Figure 14, the use of the MM improves the prediction of the viscous sublayer, which is too thin when only SSM is applied. DSM together with SSM produces the best profile. When compared with the cases with only an SGS model in

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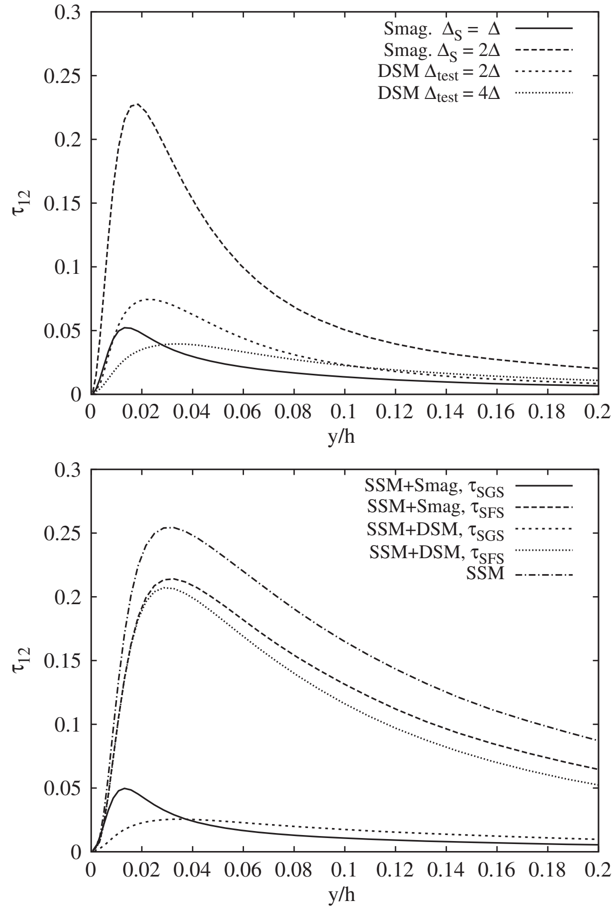

Figure 16. SGS and SFS shear stresses. Filtering provided *via* the models. Upper: Only SGS stress is modelled. Lower: SFS or SFS and SGS stresses are modelled.

the upper part of the figure, the mean bulk velocity is a bit low, but the thickness of the logarithmic layer is better predicted.

In Figure 14, the filtering provided by modelling does not change the slope of the velocity profile in the way that explicit filtering of the convection term did in Figure 10. In addition, the behaviour in the viscous sublayer improves, which does not happen in Figure 10.

The resolved deviatoric streamwise Reynolds stress from the different cases is plotted in Figure 15. In the upper part of the figure, we notice the typical behaviour of the standard Smagorinsky model as the model length scale is increased. The Reynolds stress becomes overpredicted and the distribution widens. When the dynamic model is applied, the increased test-filter width has only a small effect on the Reynolds stress and it becomes slightly more overpredicted. In the lower part of the figure, the use of SSM together with the standard Smagorinsky model produces the best Reynolds stress. However, the differences between the cases are small. By comparison of the upper and lower figures, we see that the use of SSM as an SFS model clearly

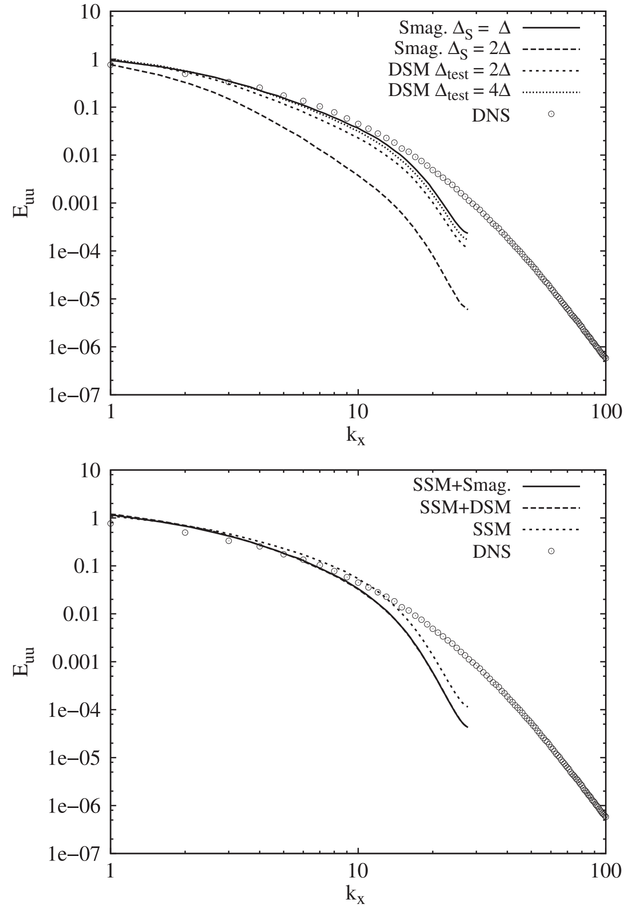

Figure 17. One-dimensional energy spectra at $y^+ \approx 5$. Streamwise direction. Filtering provided *via* the models. Upper: Only SGS stress is modelled. Lower: SFS or SFS and SGS stresses are modelled.

improved the prediction of the Reynolds stress. The results are similar for the other diagonal stress components. When compared with the case with explicit filtering of the non-linear convection term in Figure 11, the clear overprediction produced by the filtering is not visible in any of the cases in Figure 15.

In Figure 16, we have the SGS and SFS stresses from the different cases (τ_{12} of Equations (6) and (8)). As the model length scale is increased, the SGS shear stress of the standard Smagorinsky model is increased strongly. When the larger test filter is applied in DSM, the SGS shear stress inconsistently decreases. Thus, the improved results obtained using this test filter were probably caused by the decreased effect of the model. In the lower part of figure, the SFS stress produced by SSM is much larger than the SGS stress produced by the Smagorinsky models. Thus, using the SFS model increases the effect of modelling.

The one-dimensional energy spectra are depicted in the streamwise direction in Figure 17. As can be expected, increasing the length scale of the Smagorinsky model damps down the whole

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spectrum. In the lower part of the figure, the MMs affect the low frequencies less, and damp down the high frequencies better than the Smagorinsky models alone in the upper figure. However, the differences are rather small, and compared with the case with explicit filtering of the convection term in Figure 13, the damping of high frequencies is clearly not as efficient.

7. CONCLUSIONS

In this paper, we have applied explicit filtering in actual LES, studied the effect of filtering to the total simulation error, the choice of the filter function and the differences between some approaches to filtering.

Despite the promising results of the *a priori* tests [2, 5] on the numerical error, we have seen that explicit filtering of the non-linear convection term does not necessarily improve the total error of an actual simulation. On the contrary, the total simulation error clearly increased when explicit filtering with a smooth filter was applied. Some of the simulations were repeated using a finer grid to see whether the applied grid resolution was too coarse for explicit filtering (not shown here), but it did not affect the conclusion—explicit filtering still increased the total error. In Reference [10], both second- and fourth-order methods were applied, and when the grid resolutions were properly chosen, there were only small differences between the results. Since here increasing the grid resolution did not remove the negative effect of filtering, it can be assumed that it would remain also if a higher-order numerical scheme was applied.

We also studied the use of different filter functions, and the commutative filters produced slightly better results than the trapezoidal and Simpson filters. However, the negative effect of filtering remained also with the commutative filters, and despite the differences in the shapes of the filter transfer functions, the effect on the flow statistics was quite small.

By comparing the simulation results with and without filtering to simulations with no SGS model, we saw that when explicit filtering was applied, modelling had a rather small effect on the simulation results, and the main part of the increased total error was caused by explicit filtering itself. When the results with SGS modelling and filtering were compared with the simulation with filtering but no model, we also saw some improvements in the simulation results when the model length scale was increased. Thus, there was some beneficial interaction of filtering and modelling although the total simulation error still remained large. In Section 5, it was also verified that the behaviour was not due to the standard Smagorinsky model—similar results were also obtained with the dynamic version of the model and with the scale-similarity model. The effect of modelling remained small and the effect of filtering and the total simulation error large. Thus, it seems difficult to compensate for the effect of explicit filtering of the convection term performed with a smooth filter *via* SGS modelling.

Finally, we applied the approach of [9] to explicit filtering, and noticed that filtering *via* the SFS model does not have negative effect on the total error. In Reference [10], it was already shown that the use of the SFS model is necessary, and also here better results were obtained when SSM was used as an SFS model. However, we noticed that when filtering was provided by the SFS and SGS models, the small scales were not damped as strongly as when the convection term was filtered; thus, the filtering provided by the applied models is not the same as explicit limiting of the high frequencies. The use of MMs improved the prediction of the spectra compared with using Smagorinsky models or SSM alone. If one considers the total simulation error and the computing time, this latter approach seems to be better than filtering of the non-linear convection

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term. However, the original reason for using explicit filtering was the large numerical error, and in the latter approach, it is not clear if this error was decreased.

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